Partial sum processes in p-variation norm

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Let $X_1, X_2, \ldots$ be a sequence of i.i.d. real-valued random variables. For each integer $n \geq 1$, let $S_n(t) := \sum_{i=1}^{\lfloor nt \rfloor} X_i$, $t \in [0,1]$, be the n-th partial sum process. Then the partial sum process is the sequence of nth partial sum processes $S_n = \{S_n(t)\colon\ t \in [0,1]\}$, $n \geq 1$. For a function $f\colon\ [0,1] \to \mathbb{R}$ and a number $p \in (0,\infty)$, the p-variation of f is

$$v_p(f) := \sup\Big\{ \sum_{i=1}^{m} \big|f(t_i) - f(t_{i-1})\big|^p\colon\ 0 = t_0 < t_1 < \cdots < t_m = 1,\ m \in \mathbb{N}_+ \Big\}.$$

If $v_p(f) < \infty$ then f has bounded p-variation and the set of all such functions is denoted by $\mathcal{W}_p[0,1]$. For each $f \in \mathcal{W}_p[0,1]$ and $1 \leq p < \infty$, let $\|f\|_{[p]} := \|f\|_{\sup} + v_p(f)^{1/p}$, where $\|f\|_{\sup} := \sup\{|f(x)|\colon\ x \in [0,1]\}$. The set $\mathcal{W}_p[0,1]$ is a Banach space with the norm $\|\cdot\|_{[p]}$. We plan to discuss the following result obtained jointly with A. Račkauskas, as well as its extensions and applications.

Let $2 < p < \infty$ and let $W = \{W(t)\colon\ t \in [0,1]\}$ be a Wiener process. The convergence

$$n^{-1/2} S_n \Rightarrow W \quad \text{in law in} \quad \mathcal{W}_p[0,1],$$

as $n \to \infty$ holds if and only if $EX_1 = 0$ and $EX_1^2 = 1$.